* 1. Uniqueness of the limit
* 2. Sum of converging sequences
* 3. Product of converging sequences
* 4. !!! Permanence sign theorem for sequences !!!
* **Permanence of sign for continuous functions**
* 5. !!! Limit of a monotone sequence !!!
* 6. Derivative of the sum
* 7. Derivative of product
* 8. Derivate of quotient of functions
* 9. Lagrange Theorem

# 1. Uniqueness of the limit

Suppose a ≠ b

(specify a < b)

| an - a | < ε and | an - b | < ε

ε = (b - a)/2

The terms of our sequence will eventually be less than (b - a)/2 away from the limit a and (b - a)/2 away from the limit b

Solve the calculus and prove the inconsistency

# 2. Sum Of Convergent Sequences Is Convergent

Since we need to prove that {an+bn} → a+b

Rewrite as

{ (an-a) + (bn-b) } use triangle inequality

∃ N1 > n N1 € IN ∃ N2 > n N2 € IN

| an - a |< ε/2 | bn - b |< ε/2

N = max{N1 , N2 }

# 3. Product Of Convergent Sequences Is Convergent

Add and subtract abn

Triangle inequality

| bn - b | < ε/ ( 2 |a| **+1** )

∃c € IR c≥ |bn| ∀ n > N

| an - a | < ε/( 2 |c| **+1** )

# 4. Permanence sign theorem for sequences

lim an = l € [ -∞, +∞]

n

If l>0 [including +∞] than an>0 definitely, that is ∃n0 € IN such that ∀n>n0 an>0

∀ε>0 ∃n0 € IN s.t ∀n ≥ n0 we have l - ε ≤ an ≥ l + ε

Pick ε>0 such that ε=l/2 ∀ n≥n0 we have 0<l-ε<an so an≥0

In particular ∃ n0 € IN s.t. ∀n≥ n0 we have an>l/2

Then if l is positive also an is positive

# 5. Limit Of A Monotone Sequence

Prove that is a sequence is bounded and increasing that it is convergent

Consider the set A = {an | n € IN }

If it is not empty and bounded above that it has a supremum L= sup{A}

Prove that L = an

By definition of supremum: L-ε < an0 ≤ L

Fix n≥n0 so L-ε < an0 ≤ L so

L-ε < an0 < an ≤ L

L-ε < an0 < an ≤ L+ε

So L-ε < an ≤ L+ε

# 6. Derivative of the sum

Rewrite F’(X) as

Split it in two different limits

# 7. Derivative of product

Rewrite F’(x) as

Add and subtract f(x+h)g(x)

# 8. Derivative of quotient function

Rewrite F’(x) as

Add and subtract f(x) g(x)

# 9. Lagrange

Given f(x) continuous in [a,b] and derivable in (a,b) that ∃c € [a,b] such that

f’(c) = (f(b)-f(a)) / b-a

Create auxiliary function φ(x)= f(x)-kx

Rolle: φ(a)=φ(b)

find that k = (f(b)-f(a)) / b-a

Solve

# 10. Permanence of sign for continuous functions

Let f: A ⊆ IR → IR and let x0 € A. Suppose that f is continuous in x0 so:

1. if f(x0) > 0, so exists σ > 0 such that ∀ x € A with | x - x0 | < σ we have f(x) > 0
2. if f(x0) < 0, so exists σ > 0 such that ∀ x € A with | x- x0 | < σ we have f(x) < 0

Proof:

choose ε > 0 such that f(x0) - ε > 0. So exists σ > 0 such that ∀ x € A with | x-x0 | < σ we have

0 < f(x0) - ε < f(x) < f(x0) + ε